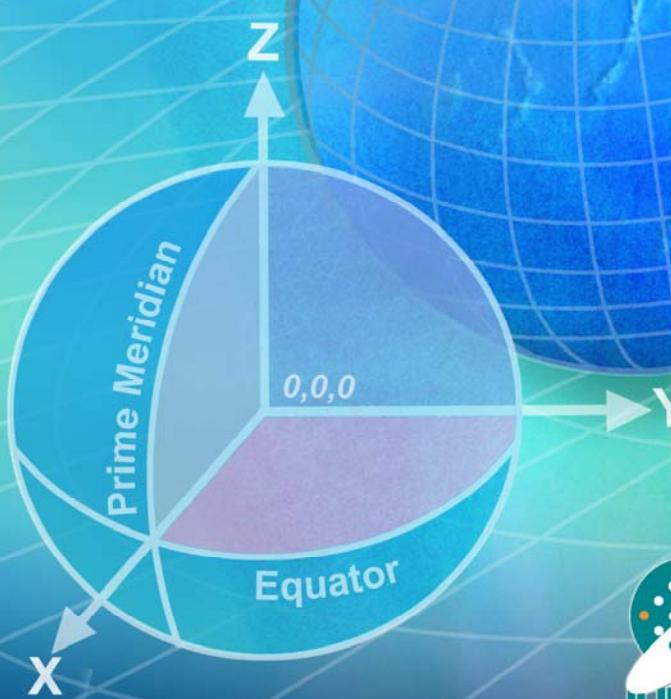


**PEKELILING
KETUA PENGARAH UKUR DAN PEMETAAN
BILANGAN 3 TAHUN 2009**

**GARIS PANDUAN MENGENAI
PENUKARAN KOORDINAT,
TRANSFORMASI DATUM
DAN UNJURAN PETA
UNTUK TUJUAN UKUR DAN PEMETAAN**



JABATAN UKUR DAN PEMETAAN MALAYSIA



Rujukan Kami: JUPEM 18/7/2.148 Jld. 3 (22)

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Semua Pengarah Ukur dan Pemetaan Negeri

PEKELILING KETUA PENGARAH UKUR DAN PEMETAAN BILANGAN 3 TAHUN 2009

GARIS PANDUAN MENGENAI PENUKARAN KOORDINAT, TRANSFORMASI DATUM DAN UNJURAN PETA UNTUK TUJUAN UKUR DAN PEMETAAN

1. TUJUAN

Pekeling ini bertujuan untuk memberikan garis panduan mengenai kaedah-kaedah penukaran koordinat, transformasi datum dan unjuran peta bagi kegunaan kerja-kerja ukur dan pemetaan.

2. LATAR BELAKANG

2.1 Jabatan Ukur dan Pemetaan Malaysia (JUPEM) bertanggungjawab melaksanakan kerja-kerja ukur hakmilik tanah dan pemetaan asas. Dalam memenuhi tanggungjawab tersebut, JUPEM telah menyediakan produk dan perkhidmatan yang berdasarkan koordinat bagi memenuhi keperluan semasa pelanggan.

2.2 Mutakhir ini teknologi satelit *Global Navigation Satellite System* (GNSS) semakin giat digunakan di dalam melaksanakan kerja-kerja ukur dan pemetaan bagi mendapatkan koordinat dengan cepat dan tepat di Malaysia.

- 2.3 Bagaimanapun, kaedah penentududukan dan penentuan koordinat berasaskan teknologi satelit GNSS telah menggunakan datum geodetik yang berlainan daripada datum-datum geodetik yang sedia ada di Malaysia. Ketidakharmonian di antara koordinat yang berpunca daripada perbezaan datum geodetik ini, jika sekiranya tidak diuruskan dengan baik, boleh menimbulkan kesilapan di dalam kerja-kerja pengukuran serta produk ukur dan pemetaan.
- 2.4 Oleh itu, maklumat mengenai kaedah penukaran koordinat, transformasi datum dan unjuran peta yang sedia ada wajar dikemaskini dan didokumentasikan supaya ianya sentiasa relevan dengan kehendak dan tuntutan semasa serta dapat dijadikan panduan dan rujukan kepada para pengguna dalam menguruskan kerja-kerja yang mempunyai kaitan dengan koordinat. Sehubungan dengan itu, Pekeliling ini diharapkan akan dapat membantu pengguna-pengguna produk-produk pemetaan, kadaster, utiliti dan sebagainya untuk memahami kaedah-kaedah tersebut dan seterusnya dapat menggunakannya dengan cara yang betul di dalam urusan kerja mereka, terutamanya di dalam era penggunaan teknologi GNSS yang semakin meluas di Malaysia.

3. PERKEMBANGAN PEMBANGUNAN INFRASTRUKTUR GEODETIK DI MALAYSIA

- 3.1 Sejak penubuhan Jabatan Ukur dan Pemetaan Malaysia (JUPEM) lebih daripada 120 tahun yang lalu, datum rujukan geodetik yang digunakan untuk kegunaan ukur dan pemetaan telah mengalami pelbagai perubahan. Dalam hal ini, sebelum tahun 1990an, Datum Kertau telah pun menjadi tulang belakang kepada *Malayan Revised Triangulation 1968* (MRT68) di Semenanjung Malaysia, manakala bagi Sabah, Sarawak dan

Labuan pula, *Borneo Triangulation 1968* (BT68) merujuk kepada Datum Timbalai sebagai asasnya.

- 3.2 Dengan perkembangan teknologi GNSS yang meluas sekitar tahun 1980-an, JUPEM telah membangunkan rangkaian kawalan ukur geodetik yang baru dengan menggunakan teknologi tersebut untuk menentukan koordinat bagi stesen-stesen kawalan. Bagi Semenanjung Malaysia, rangkaian ini telah ditubuhkan dalam tahun 1994 dan dikenali sebagai *Peninsular Malaysia Geodetic Scientific Network 1994* (PMGSN94). Manakala di Sabah, Sarawak dan Labuan pula, rangkaian kawalan ukur geodetiknya adalah *East Malaysia Geodetic Scientific Network 1997* (EMGSN97), yang ditubuhkan pada tahun 1997.
- 3.3 Antara tahun 1998 dan 2001 pula, JUPEM telah membangunkan rangkaian *Malaysia Active GPS System* (MASS) dan ini telah diikuti dengan *Malaysia Real-Time Kinematic GNSS Network* (MyRTKnet) antara tahun 2002 dan 2008. Dalam pada itu, pada 26 Ogos 2003, JUPEM telah melancarkan *Geocentric Datum of Malaysia* (GDM2000), iaitu datum rujukan geodetik baru yang seragam bagi kerja-kerja ukur dan pemetaan di Malaysia.
- 3.4 Bagaimanapun, sistem rujukan GDM2000 telah mengalami anjakan yang signifikan, lanjutan daripada berlakunya beberapa siri gempabumi besar di Indonesia, terutamanya pada tahun 2004 dan 2005. Oleh hal yang demikian, datum ini telah disemak dan dihitung semula bagi menghasilkan GDM2000 (2009).
- 3.5 Maklumat lebih lanjut mengenai sistem rujukan koordinat terkandung di dalam Pekeliling Ketua Pengarah Ukur dan Pemetaan Malaysia bilangan 1/2009 bertarikh 25 Mei 2009 bertajuk ‘Garis Panduan Mengenai Sistem Rujukan Koordinat Di dalam Penggunaan *Global Navigation Satellite System* (GNSS) Bagi Tujuan Ukur dan Pemetaan’. Ia menyenaraikan semua jenis

sistem rujukan koordinat yang telah dibangunkan oleh JUPEM di Malaysia disamping memberi maklumat-maklumat teknikal mengenai sistem tersebut dan datum geodetik.

4. GARIS PANDUAN PENUKARAN KOORDINAT, TRANSFORMASI DATUM DAN UNJURAN PETA UNTUK TUJUAN UKUR DAN PEMETAAN

Penerangan lebih lanjut tentang amalan penggunaan penukaran koordinat, transformasi datum dan unjuran peta terkandung di dalam dokumen *Technical Guide to the Coordinate Conversion, Datum Transformation and Map Projection* seperti di **Lampiran ‘A’** yang disertakan. Intisari garis panduan tersebut adalah seperti berikut:

<u>Perenggan</u>	<u>Perkara</u>
1.	<i>INTRODUCTION</i>
2.	<i>COORDINATE CONVERSION</i>
2.1	<i>GEOGRAPHICAL AND CARTESIAN COORDINATES</i>
2.2	<i>CONVERSION BETWEEN GEOGRAPHICAL COORDINATES AND CARTESIAN COORDINATES</i>
2.3	<i>TEST EXAMPLE</i>
3.	<i>DATUM TRANSFORMATION</i>
3.2	<i>INTRODUCTION</i>
3.2	<i>BURSA-WOLF DATUM TRANSFORMATION FORMULAE</i>
3.3	<i>MULTIPLE REGRESSION MODEL</i>
3.4	<i>TEST EXAMPLES</i>
4.	<i>MAP PROJECTION</i>
4.1	<i>RECTIFIED SKEW ORTHOMORPHIC PROJECTION (RSO)</i>
4.2	<i>CASSINI-SOLDNER PROJECTION</i>
4.3	<i>POLYNOMIAL FUNCTION</i>

4.4 RE-DEFINITION OF STATE ORIGINS IN GDM2000 AND
GDM2000 (2009)

4.5 TEST EXAMPLES

5. CONCLUSION

5. TARIKH BERKUATKUASA

Pekeliling ini adalah berkuatkuasa mulai tarikh ianya dikeluarkan.

Sekian, terima kasih.

“BERKHIDMAT UNTUK NEGARA”



(DATUK HAMID BIN ALI)
Ketua Pengarah Ukur dan Pemetaan
Malaysia

Salinan kepada:

Timbalan Ketua Pengarah Ukur dan Pemetaan

Pengarah Ukur Bahagian (Pemetaan)

Pengarah Ukur Bahagian (Kadaster)

Pengarah
Bahagian Geospatial Pertahanan

Setiausaha Bahagian (Tanah, Ukur dan Pemetaan)
Kementerian Sumber Asli dan Alam Sekitar

Pengarah
Institut Tanah dan Ukur Negara (INSTUN)
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Pengarah
Pusat Infrastruktur Data Geospatial Negara (MaCGDI)
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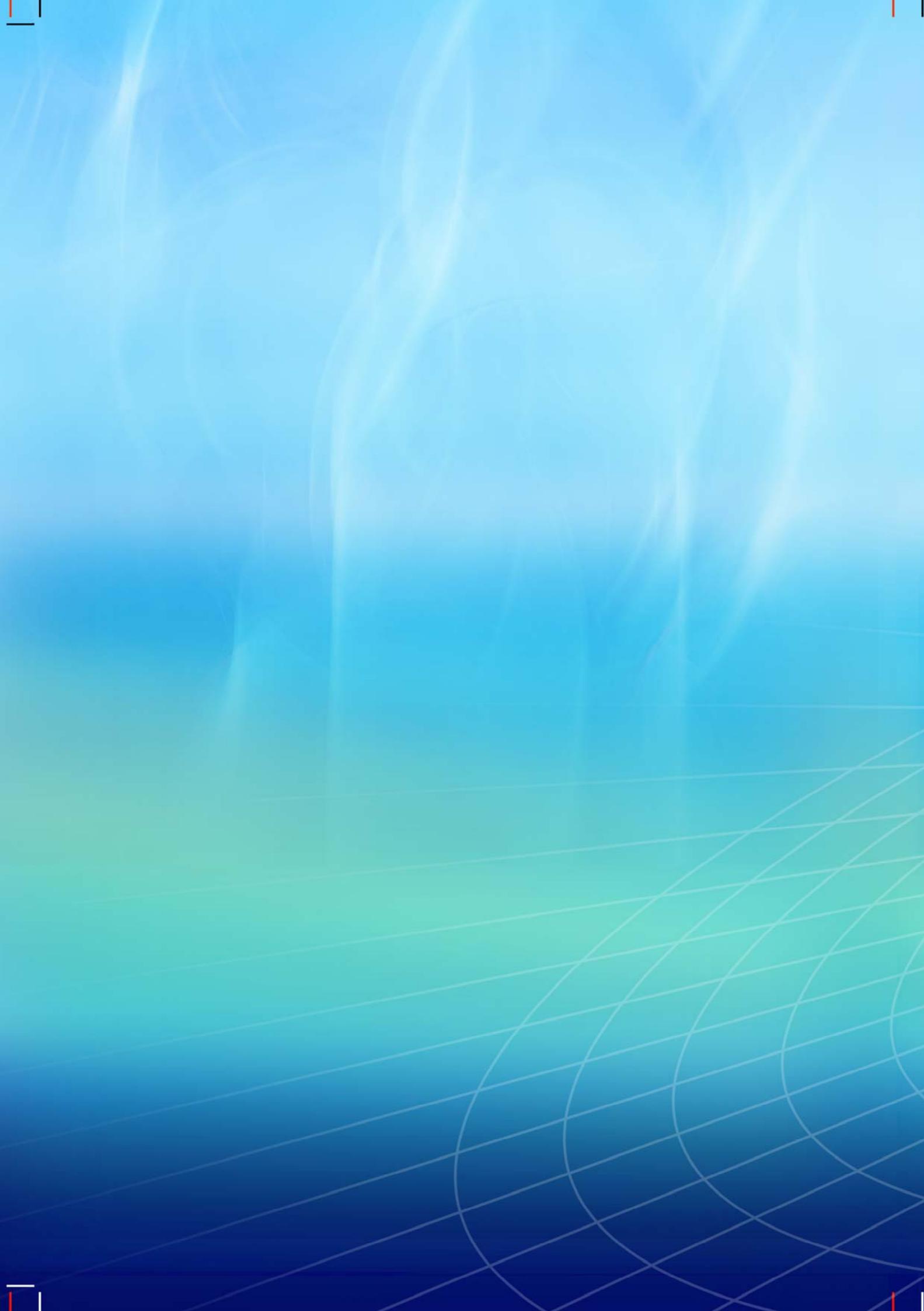
Ketua Penolong Pengarah
Unit Ukur Tanah, Cawangan Pengkalan Udara dan Maritim
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Setiausaha
Lembaga Jurukur Tanah Semenanjung Malaysia

Setiausaha
Lembaga Jurukur Tanah Sabah

Setiausaha
Lembaga Jurukur Tanah Sarawak



Technical Guide to the Coordinate Conversion, Datum Transformation and Map Projection



JABATAN UKUR DAN PEMETAAN MALAYSIA

2009

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1. INTRODUCTION

1.1 The Department of Survey and Mapping Malaysia (JUPEM) defines and maintains the Coordinate Reference System (CRS) and the Vertical Reference System (VRS) for the whole country. It establishes and manages these geodetic infrastructures for the purpose of cadastral survey, mapping, engineering and scientific research. The coordinate reference systems that have been introduced and used since the late 1800s in Malaysia are listed in Table 1.

Table 1: Coordinate Reference Systems in Malaysia

No.	Coordinate Reference System	
	Coordinate System	Geodetic Datum
1.	Malayan Revised Triangulation 1968 (MRT68)	KERTAU Ellipsoid: Modified Everest
2.	Borneo Triangulation 1968 (BT68)	TIMBALAI Ellipsoid: Modified Everest
3.	Peninsular Malaysia Geodetic Scientific Network 1994 (PMGSN94)	WGS84 Ellipsoid: WGS84 Reference Frame: WGS84 Epoch: 1987.0
4.	East Malaysia Geodetic Scientific Network 1997 (EMGSN97)	WGS84 Ellipsoid: WGS84 Reference Frame: WGS84 (G783) Epoch: 1997.0
5.	Malaysia Active GPS System (MASS)	GDM2000 Ellipsoid: GRS80 Reference Frame: ITRF2000 Epoch: 2000.0
6.	Malaysia Primary Geodetic Network 2000 (MPGN2000)	GDM2000 Ellipsoid: GRS80 Reference Frame: ITRF2000 Epoch: 2000.0
		GDM2000 (2009) Ellipsoid: GRS80 Reference Frame: ITRF2000 Epoch: 2000.0

Table 1: (continued)

No.	Coordinate Reference System	
	Coordinate System	Geodetic Datum
7.	Malaysia Real-Time Kinematic GNSS Network (MyRTKnet)	GDM2000 Ellipsoid: GRS80 Reference Frame: ITRF2000 Epoch: 2000.0
		GDM2000 (2009) Ellipsoid: GRS80 Reference Frame: ITRF2000 Epoch: 2000.0

- 1.2 Figure 1 represents a schematic diagram to assist users in navigating between the different types of coordinates. All Coordinate Reference Systems (CRS) available in Malaysia are shown; some are on the same geodetic datum and others, on different ones. Coordinate conversions which do not involve a change of datum are shown as dashed lines. Datum transformations, on the other hand, involve a change of datum and are shown as thick solid lines and map projection as thin solid lines.
- 1.3 This technical guide is produced to assist users in understanding the concept and procedures involved in the process of coordinate conversion, datum transformation and map projection as practiced in Malaysia.

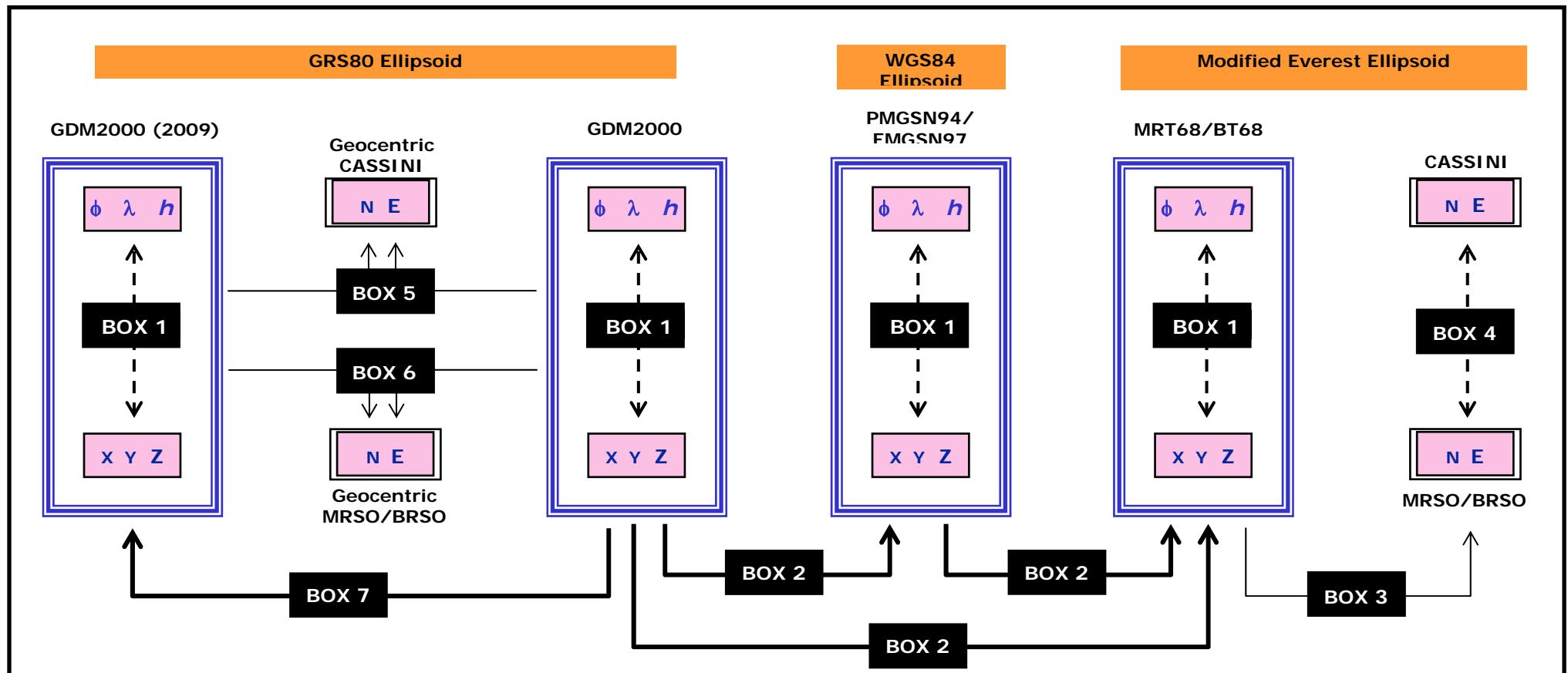


Figure 1: Relationship between various coordinate reference system

Notes to Figure 1:

- Box 1:** Coordinate conversion between geographical coordinate and cartesian coordinate
- Box 2:** Transformation between various datums using Bursa-Wolf formulae
- Box 3:** Map projection of MRT68 / BT68 geographical coordinate to Rectified Skew Orthomorphic (RSO) plane coordinates
- Box 4:** Coordinate transformation from RSO to Cassini using polynomial function
- Box 5:** Map projection of GDM2000 geographical coordinate to Geocentric Cassini plane coordinates
- Box 6:** Map projection of GDM2000 geographical coordinate to Geocentric RSO plane coordinates
- Box 7:** Datum transformation from GDM2000 to GDM2000 (2009) using multiple regression model

2. COORDINATE CONVERSION

2.1 Geographical and Cartesian Coordinates

2.1.1 Three-dimensional geographical coordinates can be defined with respect to an ellipsoid as follows:

Latitude: the angle north or south from the equatorial plane

Longitude: the angle east or west from the prime meridian

Height: the distance above the surface of the ellipsoid.

2.1.2 A set of cartesian coordinates is defined with the three axes at the origin at the center of the ellipsoid, such that:

Z-axis: is aligned with the minor (or polar) axis of the ellipsoid

X-axis: is in the equatorial plane and aligned with the prime meridian

Y-axis: forms a right-handed system

2.1.3 In this regard, positions in geographical coordinates of latitude, longitude and height (ϕ , λ , h) can be converted into cartesian coordinates (X, Y, Z) and vice-versa.

Geographical Coordinates	Cartesian Coordinates
Latitude, Longitude and Height (ϕ , λ , h)	X Y Z

2.2 Conversion between Geographical Coordinates and Cartesian Coordinates

2.2.1 The conversion of three-dimensional coordinates from geographical to cartesian or vice versa can be carried out through the knowledge of the parameters of an adopted reference ellipsoid (Figure 2). The

forward conversion from geodetic coordinates (ϕ, λ, h) to cartesian coordinate (X, Y, Z) is as follows:

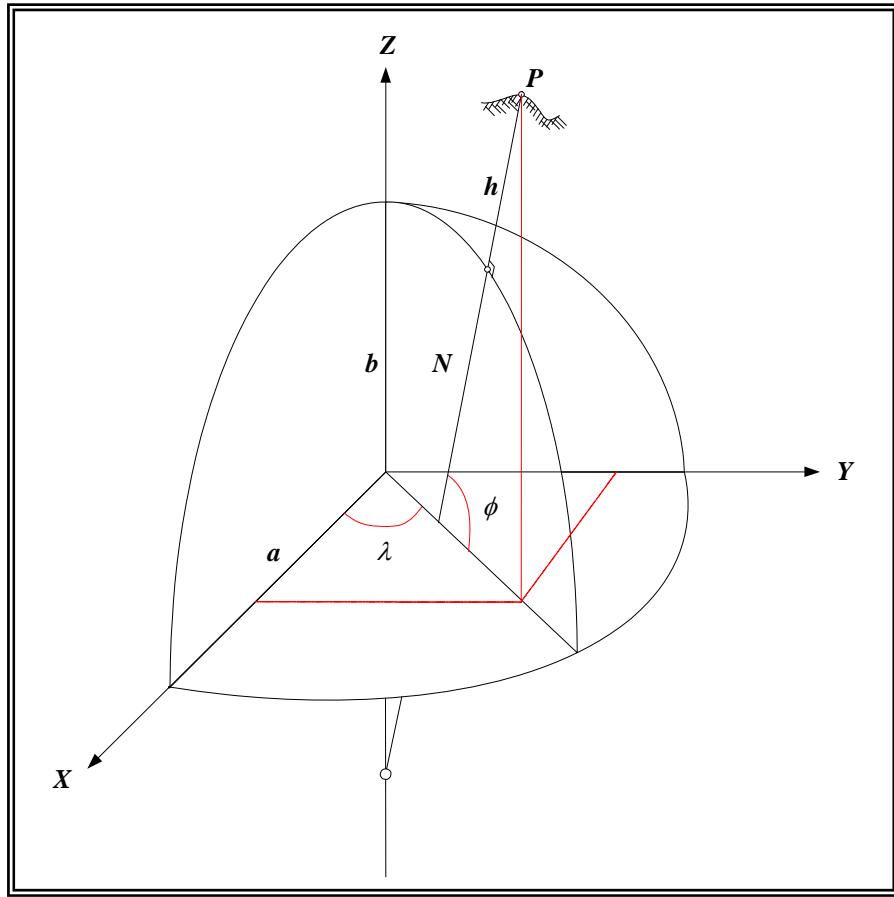


Figure 2: Geographical and cartesian coordinates

$$X = (N + h) \cos \phi \cos \lambda$$

$$Y = (N + h) \cos \phi \sin \lambda$$

$$Z = \left(\frac{b^2}{a^2} N + h \right) \sin \phi$$

where the prime vertical radius of curvature (N) is:

$$N = \frac{a^2}{\sqrt{\frac{1}{(a^2 \cos \phi + b^2 \sin^2 \phi)^2}}}$$

with:

a : the semi-major axis of the reference ellipsoid;

b : the semi-minor axis of the reference ellipsoid;

2.2.2 The non-iterative reverse conversion from cartesian coordinates

(X, Y, Z) to geodetic coordinates (ϕ, λ, h) is as follows:

$$\phi = \arctan \left[\frac{Z + \varepsilon^2 b \sin^3 u}{P - e^2 a \cos^3 u} \right]$$

$$\lambda = \arctan \left[\frac{Y}{X} \right]$$

$$h = P \cos \phi + Z \sin \phi - a \sqrt{1 - e^2 \sin^2 \phi}$$

with:

$$u = \arctan \left[\frac{aZ}{bP} \right]$$

$$P = \sqrt{X^2 + Y^2}$$

$$\varepsilon = \frac{e^2}{1 - e^2}$$

$$e^2 = \frac{a^2 - b^2}{a^2}$$

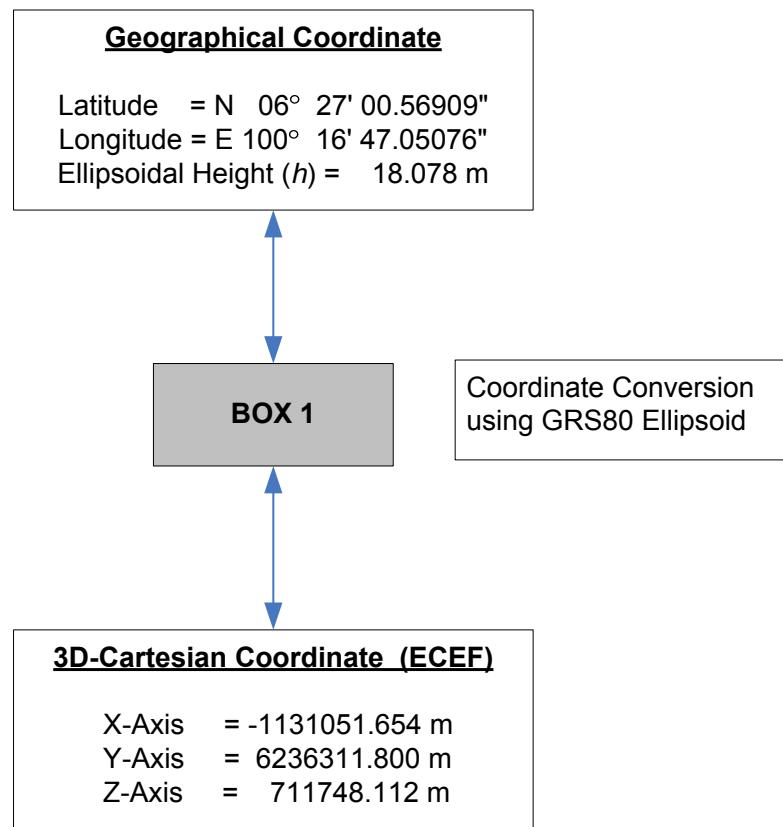
where,

u : the parametric latitude;

e : the first eccentricity of the reference ellipsoid;

ε : the second eccentricity of the reference ellipsoid.

2.3 Test Example



3. DATUM TRANSFORMATION

3.1 Introduction

3.1.1 Datum transformation is a computational process of converting a position given in one coordinate reference system into the corresponding position in another coordinate reference system. It requires and uses the parameters of the transformation and the ellipsoids associated with the source and target coordinate reference systems. For example, the source can be coordinate reference system 1 and the target can be coordinate reference system 2:

No.	Coordinate Reference System 1	Coordinate Reference System 2
1	Geocentric Datum of Malaysia (GDM2000)	Geocentric Datum of Malaysia GDM2000 (2009)
		Peninsular Malaysia Geodetic Scientific Network 1994 (PMGSN94)
		East Malaysia Geodetic Scientific Network 1997 (EMGSN97)
		Malayan Revised Triangulation 1968 (MRT68)
		Borneo Triangulation 1968 (BT68) (Sabah)
		Borneo Triangulation 1968 (BT68) (Sarawak)
2	Peninsular Malaysia Geodetic Scientific Network 1994 (PMGSN94)	Malayan Revised Triangulation 1968 (MRT68)
3	East Malaysia Geodetic Scientific Network 1997 (EMGSN97)	Borneo Triangulation 1968 (BT68) (Sabah)
		Borneo Triangulation 1968 (BT68) (Sarawak)

- 3.1.2 The transformation parameter values associated with the transformation can be determined empirically from a measurement or a calculation process. The parameters are computed based on coordinates of control stations which are common to different datums. They are generated through least square analysis using various models accepted by the global geodetic community.
- 3.1.3 Datum transformation can be accomplished by many different methods. A simple three parameter conversion can be accomplished by conversion through Earth-Centred Earth Fixed (ECEF) cartesian coordinates from one reference datum to another by three origin offsets that approximate differences in rotation, translation and scale. A complete datum conversion is usually based on seven parameter transformations, which include three translation parameters, three rotation parameters and a scale.

3.2 Bursa-Wolf Datum Transformation Formulae

- 3.2.1 Bursa-Wolf formulae is a seven-parameter model for transforming three-dimensional cartesian coordinates between two datums (see Figure 3). This transformation model is more suitable for satellite datums on a global scale (Krakwisky and Thomson, 1974). The transformation involves three geocentric datum shift parameters ($\Delta X, \Delta Y, \Delta Z$), three rotation elements (R_X, R_Y, R_Z) and a scale factor ($1+\Delta L$).
- 3.2.2 The model in its matrix-vector form could be written as (Burford 1985):

$$\begin{bmatrix} X_{WGS84} \\ Y_{WGS84} \\ Z_{WGS84} \end{bmatrix} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} 1+\Delta L & R_Z & -R_Y \\ -R_Z & 1+\Delta L & R_X \\ R_Y & -R_X & 1+\Delta L \end{bmatrix} \begin{bmatrix} X_{MRT} \\ Y_{MRT} \\ Z_{MRT} \end{bmatrix}$$

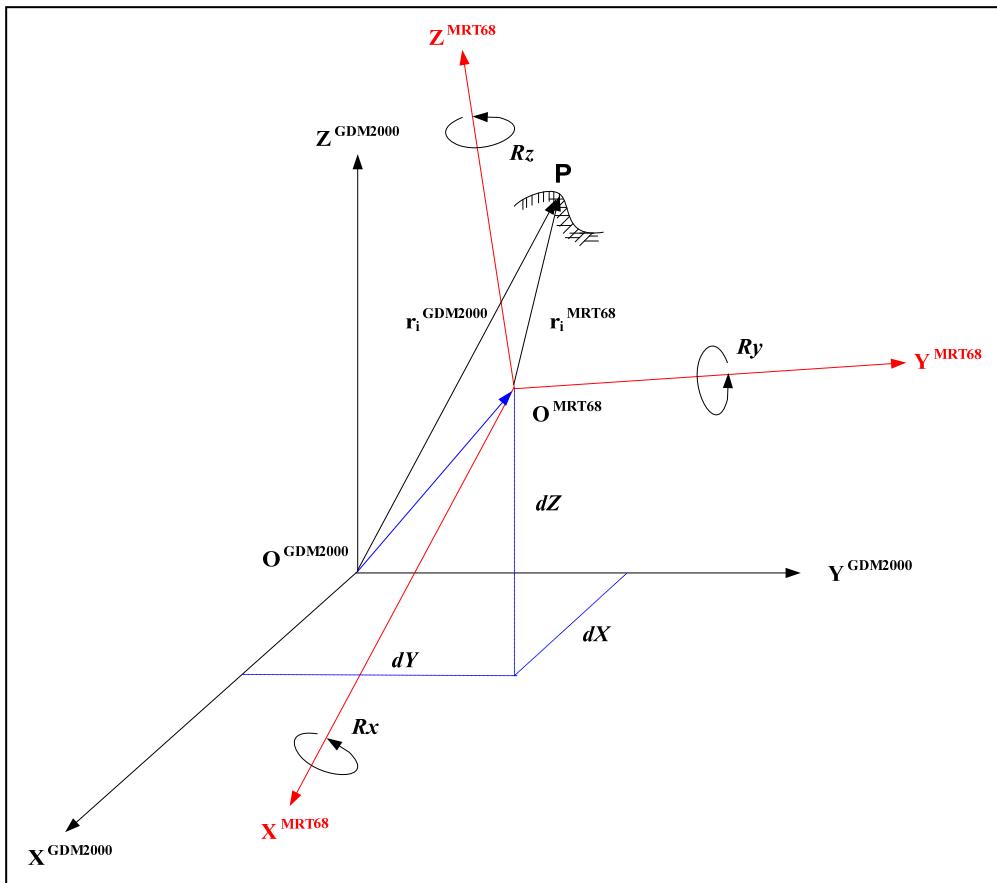


Figure 5: Bursa-Wolf 3-D model transformation

where;

$X_{WGS84}, Y_{WGS84}, Z_{WGS84}$: are the global datum (WGS84) cartesian coordinates;

$X_{MRT}, Y_{MRT}, Z_{MRT}$: are the local datum (MRT) cartesian coordinates.

In order to convert the cartesian coordinates of XYZ to the geographical coordinates of $\phi \lambda h$, ellipsoid properties for the respective datum as listed in Table 2 below are used:

Table 2: Ellipsoid Properties

No.	Ellipsoid	semi-major axis, a (m)	flattening, f (m)
1	Geodetic Reference System 1980 (GRS80)	6378137.000	298.2572221
2	World Geodetic System 1984 (WGS84)	6378137.000	298.2572236
3	Modified Everest (Peninsular Malaysia)	6377304.063	300.8017
4	Modified Everest (East Malaysia)	6377298.556	300.8017

3.3 Multiple Regression Model

(i) Displacement Computation

The computation and modelling of differences in the coordinate between the two systems, i.e. GDM2000 and GDM2000 (2009) were carried out using their respective geographical coordinates in the format of (ϕ , λ and h). These differences are then converted to the local geodetic horizon to avoid mathematical errors for some very small values. The conversion from geographical system to local geodetic system used the following factor, i.e. $1'' = 30$ meter.

The differences in the three components are computed separately by using the following formulae:

$$\Delta \text{North (N)} = (\phi_{\text{GDM2000}} - \phi_{\text{GDM2000 (2009)}}) \times 30$$

$$\Delta \text{East (E)} = (\lambda_{\text{GDM2000}} - \lambda_{\text{GDM2000 (2009)}}) \times 30$$

$$\Delta \text{Height (U)} = (h_{\text{GDM2000}} - h_{\text{GDM2000 (2009)}})$$

(ii) Displacement Modelling

The differences in coordinate of every GPS station are similarly computed between the two systems, i.e. GDM2000 and GDM2000 (2009) using their respective coordinates in the format of (E and N). The coordinate differences are then gridded to derive the Regression Coefficient. The gridding method used is the polynomial regression with the power to the second order. The surface definition used the bi-linear saddle regression coefficient with the following formulae:

$$Z(E, N, U) = A_{00} + A_{01} N + A_{10} E + A_{11} EN$$

where,

Z = Value of regression coefficient or the value of displacement correction for each component (i.e. East, North and Up)

E = East coordinate in decimal degree

N = North coordinate in decimal degree

(iii) Basic Formulae

To convert the coordinate in GDM2000 to GDM2000 (2009), the following formulae shall be used:

$$\text{GDM2000 (2009)} = \text{GDM2000} + \text{correction}$$

$$\phi''_{\text{GDM2000 (2009)}} = \phi''_{\text{GDM2000}} + (Z_N / 30)$$

$$\lambda''_{\text{GDM2000 (2009)}} = \lambda''_{\text{GDM2000}} + (Z_E / 30)$$

$$h_{\text{GDM2000 (2009)}} = h_{\text{GDM2000}} + Z_U$$

where,

ϕ	=	Latitude in second of arc
λ	=	Longitude in second of arc
h	=	Ellipsoidal height in metre
Z_N	=	Displacement correction in northing
Z_E	=	Displacement correction in easting
Z_U	=	Displacement correction in height

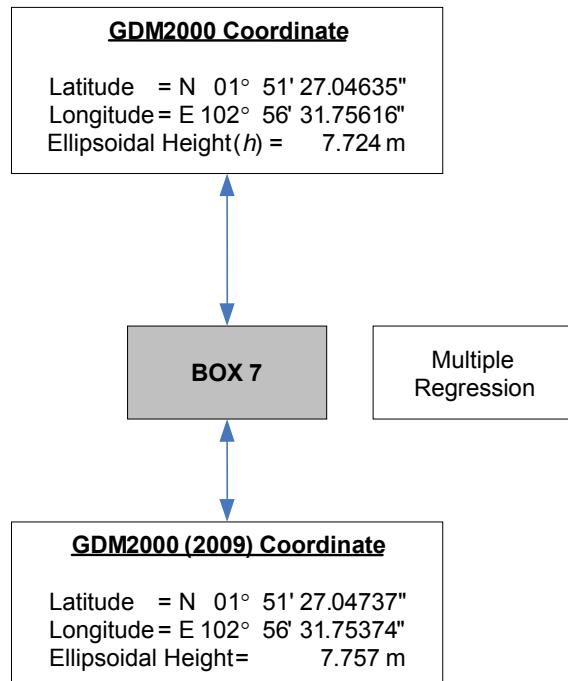
where,

$$Z(E, N, U) = A_{00} + A_{01} N + A_{10} E + A_{11} EN$$

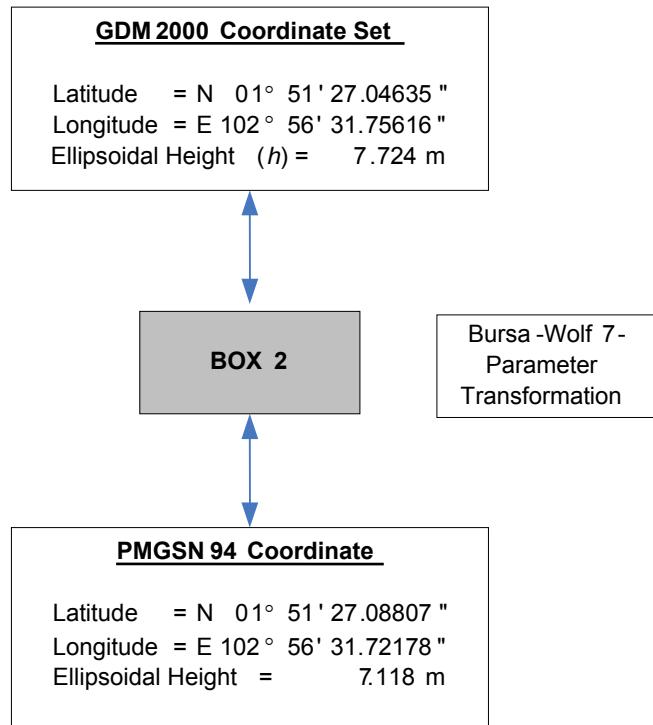
and A_{00} , A_{01} , A_{10} and A_{11} are the coefficients of the multiple regression model.

3.4 Test Examples

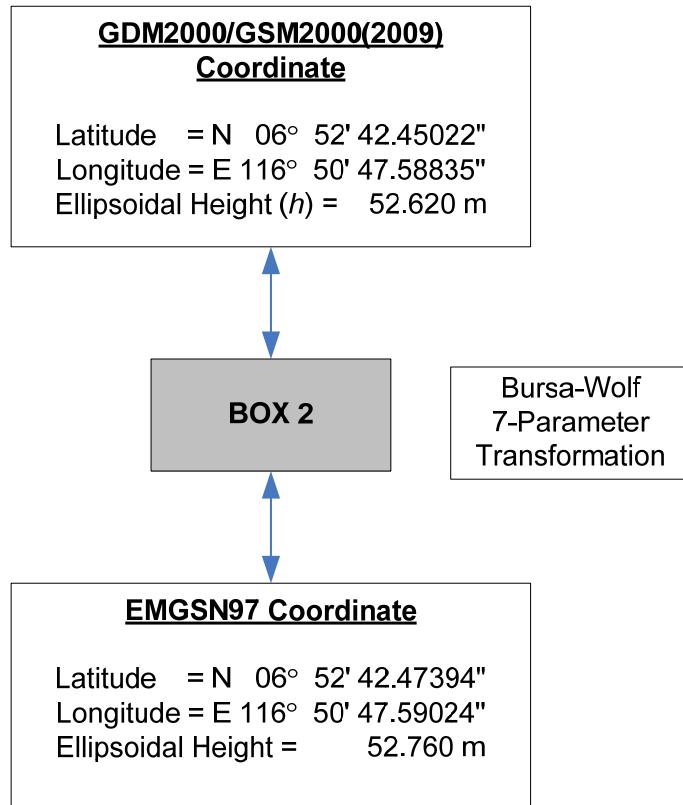
(i) GDM2000 to GDM2000 (2009)



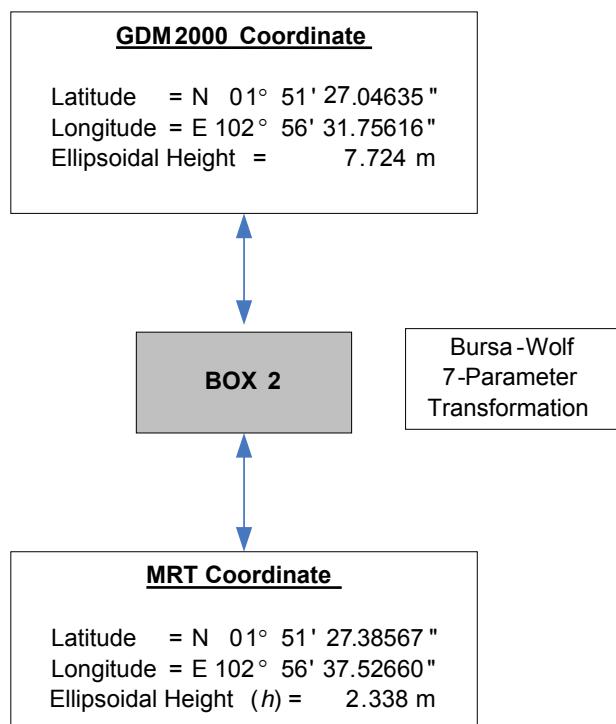
(ii) GDM2000 to PMGSN94



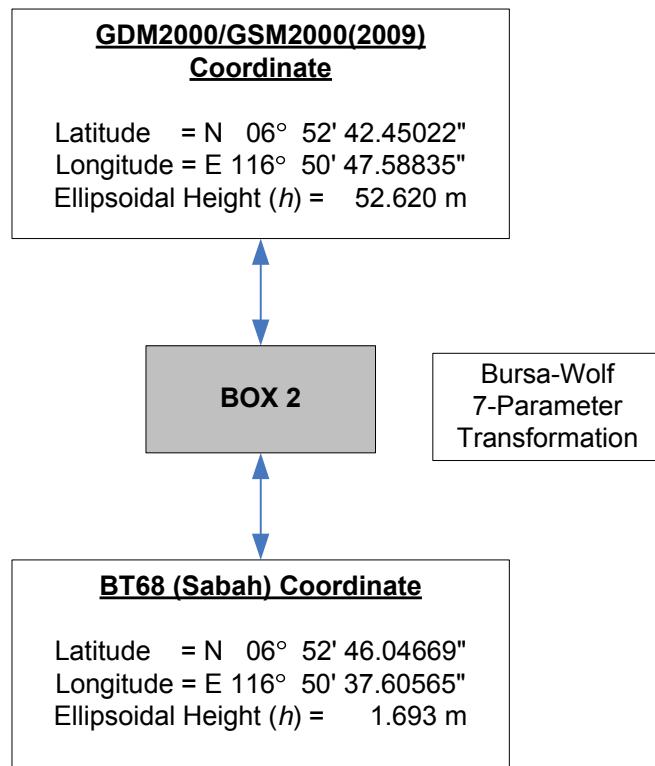
(iii) GDM2000 / GDM2000 (2009) to EMGSN97



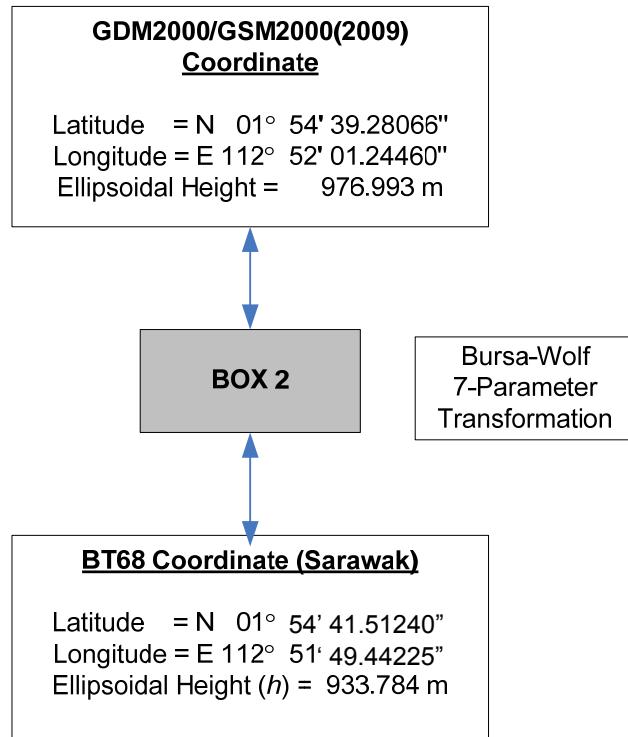
(iv) GDM2000 to MRT68



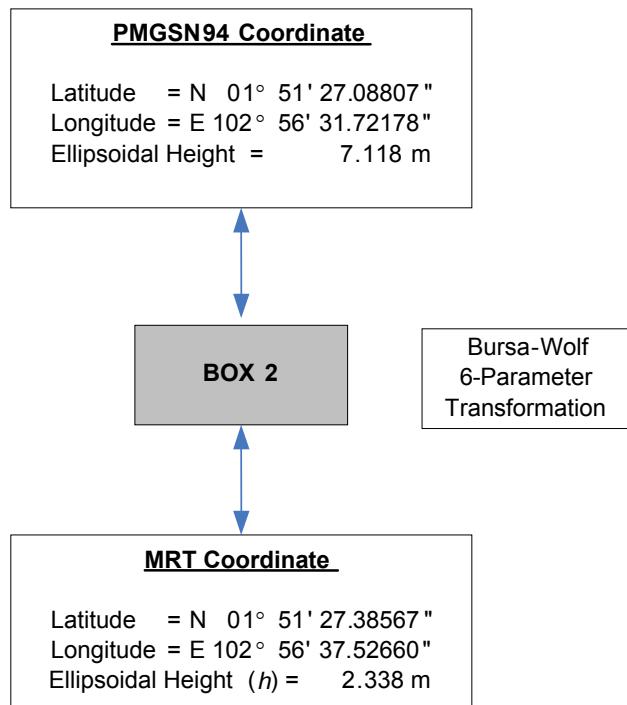
(v) GDM2000 / GDM2000 (2009) to BT68 (Sabah)



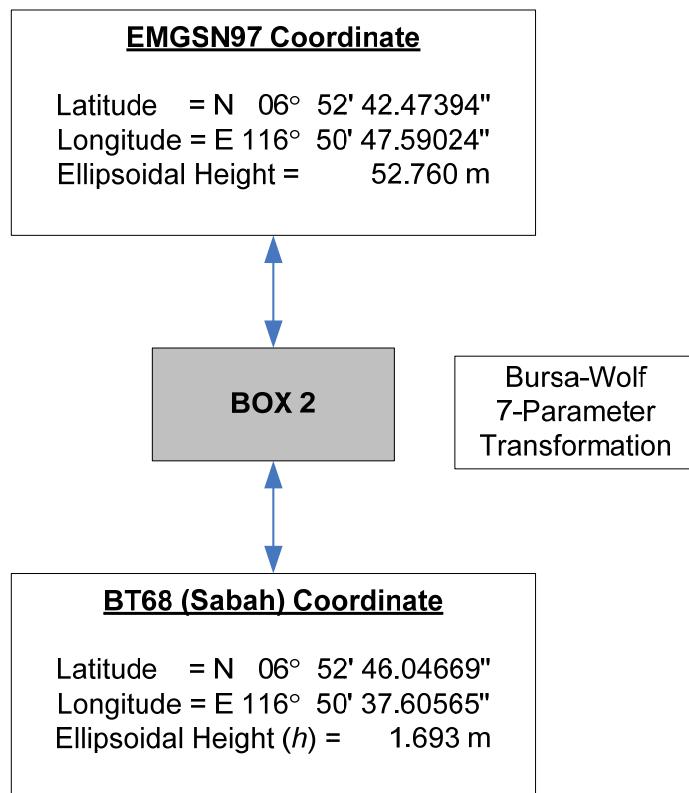
(vi) GDM2000 / GDM2000 (2009) to BT68 (Sarawak)



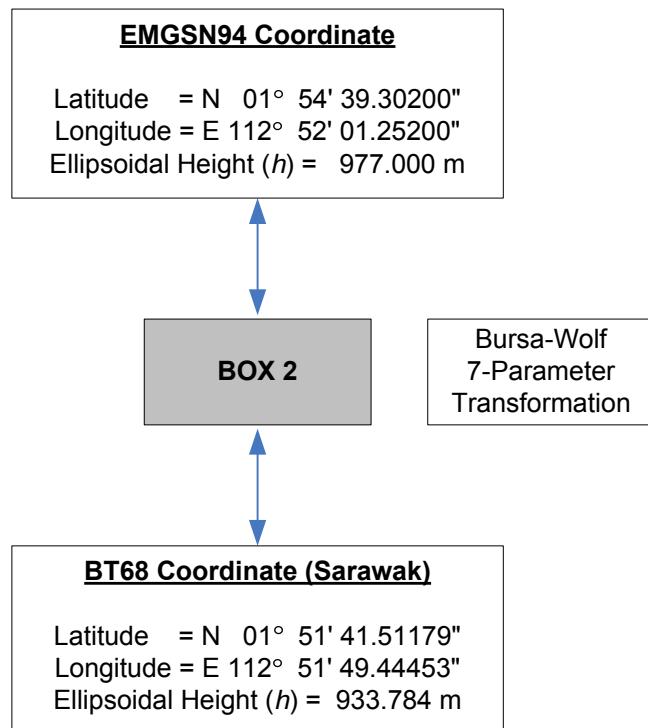
(vii) PMGSN94 to MRT68



(viii) EMGSN97 to BT68 (Sabah)



(ix) EMGSN97 to BT68 (Sarawak)



4. MAP PROJECTION

4.1 Introduction

4.1.1 Typically, there are many different methods for projecting longitude and latitude in coordinate reference system 1 onto a flat map in the same system:

No.	Coordinate Reference System 1	Map Projection System 1
1	Geocentric Datum of Malaysia GDM2000 (2009) Geocentric Datum of Malaysia (GDM2000)	Geocentric Cassini-Soldner
2	Geocentric Datum of Malaysia GDM2000 (2009) Geocentric Datum of Malaysia (GDM2000)	Geocentric Malayan Rectified Skew Orthomorphic (Peninsular Malaysia)
3	Geocentric Datum of Malaysia GDM2000 (2009) Geocentric Datum of Malaysia (GDM2000)	Geocentric Borneo Rectified Skew Orthomorphic (Sabah and Sarawak)
4	Malayan Revised Triangulation 1968 (MRT68)	Malayan Rectified Skew Orthomorphic (Peninsular Malaysia)
5	Rectified Skew Orthomorphic (Peninsular Malaysia)	Cassini-Soldner _{Old}
6	Borneo Triangulation 1968 (BT68)	Borneo Rectified Skew Orthomorphic (Sabah and Sarawak)

4.2 Rectified Skew Orthomorphic (RSO) Map Projection

4.2.1 The rectified skew orthomorphic (RSO) map projection (Figure 4) is an oblique Mercator projection developed by Hotine in 1947 (Snyder, 1984). This projection is orthomorphic (conformal) and cylindrical. All meridians and parallel are complex curves. Scale is approximately

true along a chosen central line (exactly true along a great circle in its spherical form). It is thus a suitable projection for an area like Switzerland, Italy, New Zealand, Madagascar and Malaysia as well.

- 4.2.2 The RSO provides an optimum solution in the sense of minimizing distortion whilst remaining conformal for Malaysia. Table 3 tabulates the new geocentric RSO parameters for Peninsular Malaysia and East Malaysia.

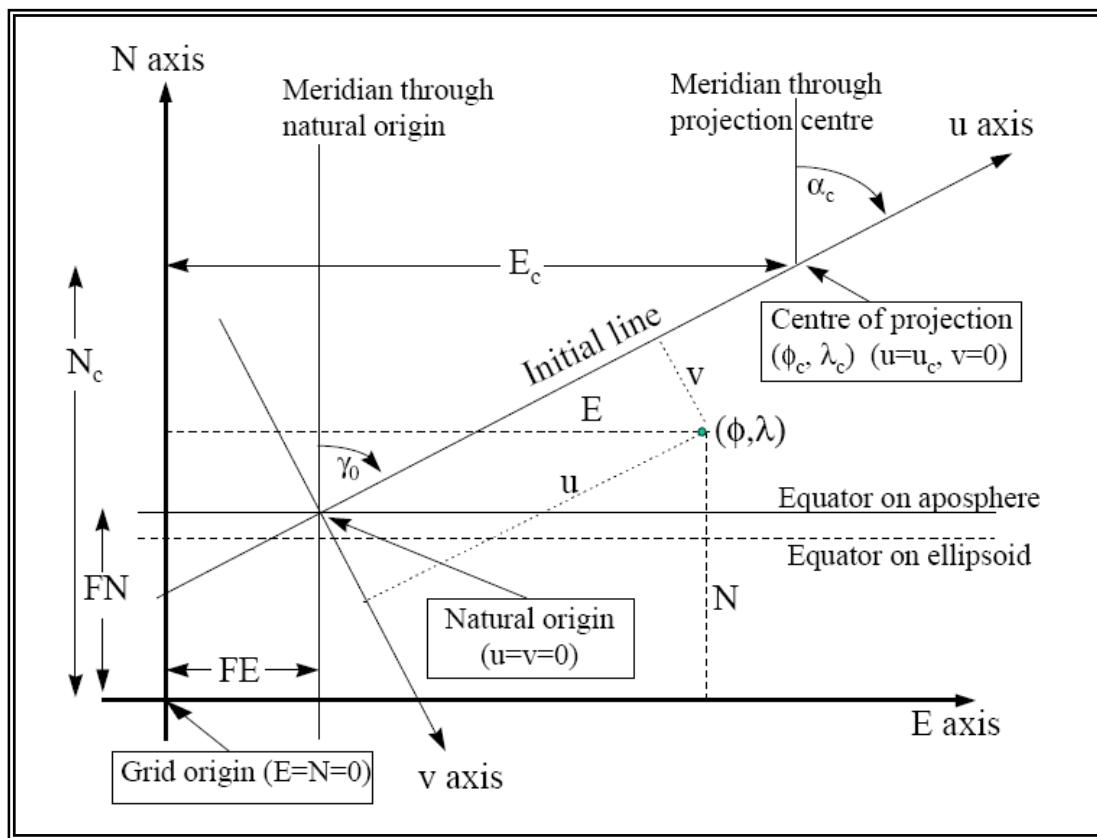


Figure 4: Hotine, 1947 (Snyder, 1984), Oblique Mercator (Source: EPSG)

Table 3: The New Geocentric RSO Projection Parameters

	Peninsular RSO	East Malaysia BRSO
Ellipsoid Parameters		
Ellipsoid	GRS 80	GRS 80
Semi Major axis, a	6378137.000 Meters	6378137.000 Meters
Flattening, $1/f$	298.2572221	298.2572221
Defined Parameters.		
Latitude of Origin, ϕ_o		
Longitude of Origin, λ_o		
Rectified to Skew Grid, γ_o		
Azimuth of Central Line, α_c		
Scale factor, k		
False Origin (Easting)	Defined projection parameters can be obtained from JUPEM	
False Origin (Northing)		

4.2.3 The notation adopted for use in this section is as follows:

ϕ_c = latitude of center of the projection.

λ_c = longitude of center of the projection.

α_c = azimuth (true) of the center line passing through the center of the projection.

γ_c = rectified bearing of the center line.

k_c = scale factor at the center of the projection.

ϕ = geographical latitude

λ = geographical longitude

t_o = isometric latitude for $\phi = 4^\circ$

$$t_o = \log \theta \tan \left(\frac{\pi}{4} + \frac{\phi_c}{2} \right) - \frac{k}{2} \log \theta \frac{1 + e \sin \phi_c}{1 - e \sin \phi_c}$$

or

$$t_0 = \frac{\tan(\frac{\pi}{4} - \frac{\phi}{2})}{[(1 - e \sin \phi)/(1 + e \sin \phi)]^{e/2}}$$

t = isometric latitude

λ_0 = basic longitude

a = semi major axis of ellipsoid

b = semi minor axis of ellipsoid

f = flattening of ellipsoid

$$f = (a - b)/a$$

e = eccentricity of ellipsoid

$$e^2 = (a^2 - b^2)/a^2$$

$$e_1^2 = (a^2 - b^2)/b^2$$

ρ = radius of curvature in the meridian

$$\rho = a(1 - e^2) / (1 - e^2 \cdot \sin^2 \phi)^{3/2}$$

ν = radius of curvature in the prime vertical

$$\nu = \rho(1 + e_1^2 \cdot \cos^2 \phi)$$

m = scale factor

m_o = scale factor at the origin

γ = skew convergence at meridians

p = distance from polar axis, $p = \nu \cos \varphi$

γ_R = rectified convergence of meridians

$$\gamma_R = \gamma + 36^\circ 52' 11.6314''$$

u = skew coordinate parallel to initial line

v = skew coordinate at right angles to initial line

N = Northing map coordinate

E = East map coordinate

FE = False Easting at the natural origin.

FN = False Northing at the natural origin.

4.2.4 The Constants of the Projection (Table 4) are given as follows:

$$B = \left(1 - e_1^2 \cdot \cos^4 \phi_c\right)^{1/2}$$

or

$$B = [1 + e^2 \cos^4(\phi_c) / (1 - e^2)]^{0.5}$$

$$A = a \cdot B \cdot k_c (1 - e^2)^{0.5} / (1 - e^2 \sin^2(\phi_c))$$

$$A' = B(\rho_o v_o)^{1/2}$$

$$C = \cosh^{-1}\left(\frac{A}{p_o}\right) - Bt_o$$

$$D = \frac{B\sqrt{1-e^2}}{\cos \phi_c \cdot \sqrt{1-e^2 \sin^2 \phi_c}}$$

To avoid problems with computation of F, if D<1, make D² = 1.

$$F = D + \operatorname{sgn}(\phi_c) \cdot \sqrt{Dsq - 1}$$

$$H = F \cdot t_0^B$$

$$G = \frac{F - 1/F}{2}$$

$$\sin \gamma_o = -0.6 \text{ Or}$$

$$\gamma_0 = \arcsin[\sin(\alpha_c / D)]$$

Basic Longitude:

$$\lambda_b = \lambda - \frac{\arcsin(G \cdot \tan \gamma_0)}{B}$$

Table 4: Constant of Projection

Constant of Projection		
	Peninsular Malaysia	East Malaysia
Parameter A		
Parameter A'		
Parameter B		
Parameter C		
Basic Longitude. λ_0	Defined projection parameters can be obtained from JUPEM	

4.2.5 The projection formulae are as follows:

(a) Conversion of Geographical to Rectangular and vice versa

Forward Case: To compute (E, N) from a given (ϕ, λ):

$$t = \frac{\tan\left(\frac{\pi}{4} - \frac{\phi}{2}\right)}{[(1 - e \sin \phi) / (1 + e \sin \phi)]^{e/2}}$$

$$Q = H/t^2$$

$$S = \frac{Q - 1/Q}{2}$$

$$T = \frac{Q + 1/Q}{2}$$

$$V = \sin[B(\lambda - \lambda_0)]$$

$$U = \frac{S \sin \lambda_0 - V \cos \lambda_0}{T}$$

$$v = \frac{A \ln\left(\frac{1-U}{1+U}\right)}{2 \cdot B}$$

For the Hotine Oblique Mercator (where the FE and FN values have been specified with respect to the origin of the (u, v) axes):

$$u = \frac{A}{B} \arctan \left(\frac{S \cos \gamma_0 + V \sin \gamma_0}{\cos[B(\lambda - \lambda_0)]} \right)$$

The rectified skew coordinates are then derived from:

$$E = v \cos \gamma_c + u \sin \gamma_c + (FE \text{ or } E_c)$$

$$N = u \cos \gamma_c - v \sin \gamma_c + (FN \text{ or } N_c)$$

Reverse Case: Compute (ϕ, λ) from a given (E, N) :

For the Hotine Oblique Mercator:

$$v' = (E - FE) \cos \gamma_c - (N - FN) \sin \gamma_c$$

$$u' = (N - FN) \cos \gamma_c + (E - FE) \sin \gamma_c$$

Then the other parameters can be calculated.

$$Q' = \exp[-(Bv'/A)]$$

$$S' = \frac{Q' - 1/Q'}{2}$$

$$T' = \frac{Q' + 1/Q'}{2}$$

$$V' = \sin \left(\frac{B \cdot u'}{A} \right)$$

$$U' = \frac{V' \cos \gamma_c + S' \sin \gamma_c}{T'}$$

$$t' = \left[\frac{H}{\sqrt{(1+U')/(1-U')}} \right]^{1/B}$$

$$\chi = \pi / 2 - 2 \cdot \arctan(t')$$

$$\ell^o = \lambda + \sin(2\lambda) \left(e^2 / 2 + 5e^4 / 24 + e^6 / 12 + 13e^8 / 360 \right) + \\ \sin(4\lambda) \left(7e^4 / 48 + 29e^6 / 240 + 811e^8 / 11520 \right) + \\ \sin(6\lambda) \left(7e^6 / 120 + 81e^8 / 1120 \right) + \\ \sin(8\lambda) \left(4279e^8 / 161280 \right)$$

$$\lambda = \lambda_0 - \arctan \left(\frac{S' \cos \gamma_e - V' \sin \gamma_e}{\cos(Bu'/A)} \right) / B$$

(b) Convergence of Map Meridians

The convergence of the map meridians is defined as the angle measured clockwise from True North to the Rectified Grid North, and is denoted γ_R (Figure 5):

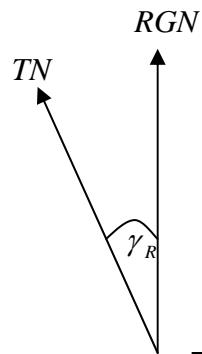


Figure 5: Convergence of Map Meridians

$$\begin{aligned} \gamma_R &= \gamma - \sin^{-1}(-0.6) \\ &= \gamma + 36^\circ 52' 11.6314'' \end{aligned}$$

where

$$\begin{aligned} \tan \gamma &= \frac{\tan \gamma_o - \sin B(\lambda_o - \lambda) \sinh(Bt + C)}{\cos B(\lambda_o - \lambda) \cosh(Bt + C)} \\ &= \frac{\sin \frac{Bv}{A'm_o} \sinh \frac{Bu}{A'm} + \tan \gamma_o}{\cos \frac{Bv}{A'm_o} \cosh \frac{Bu}{A'm_o}} \end{aligned}$$

(c) Scale Factor at any Point

The formulas giving the scale factor “ m ” at any point in terms of isometric latitude and longitude and coordinate (v,u) are:

$$m = \frac{A'm_o}{p} \frac{\cosh \frac{Bu}{A'm_o}}{\cosh(Bt + C)}$$

$$= \frac{A'm_o}{p} \frac{\cos \frac{Bv}{A'm_o}}{\cos B(\lambda_o - \lambda)}$$

It can be shown that the initial line of the projection has a scale factor that is nearly constant throughout its length.

(d) Scale Factor for a Line

The Scale factor for a line can be computed from the formula:

$$m = \frac{1}{6}(m_1 + 4m_3 + m_2)$$

where m_1, m_2 are the scale factors at the ends of the line and m_3 the scale factor at its mid-point. The scale factor for a line also may be evaluated from the following formula:

$$m = \frac{A'm_o}{v_m \cos \varphi_m \cosh(B\varphi_m + C)} \left[1 + \frac{B^2}{6A'^2 m_o^2} (u_1^2 + u_1 u_2 + u_2^2) \right]$$

where

φ_m and ψ_m are evaluated for the mid-latitude of the line
 u_1 and u_2 are the u - coordinate of the points:

(e) Arc-to-Chord Correction

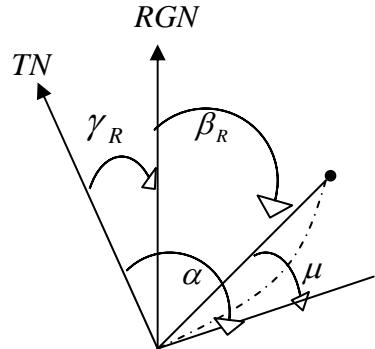


Figure 8: Arc-to-Chord

In Figure 6, if α is the true azimuth of a line, β_R is the rectified grid bearing, then

$$\alpha = \beta_R + \gamma_R$$

where μ is given in seconds by the formula

$$\begin{aligned} \mu = & \frac{B}{2A'm_o \sin 1''} (v_2 - v_1) \tanh \left\{ \frac{B}{2A'm_o} \frac{2u_1 + u_2}{3} \right\} \\ & + k_1^2 \frac{\rho_o}{v_o} \sin \phi_o (\sin \phi_3 - \sin \phi_o)^2 (\lambda_1 - \lambda_2) \end{aligned}$$

where,

$$\phi_3 = \frac{1}{3}(2\phi_1 + \phi_2) \text{ and } (\lambda_1 - \lambda_2) \text{ is measured in seconds.}$$

For a line not exceeding 113 km (70 miles) in length, the maximum value of the second term of the formula is 0.007"; it can therefore be safely neglected.

4.3 Cassini-Soldner Map Projection

- 4.3.1 There are nine state origins used in the coordinate projection in the cadastral system of Peninsular Malaysia. The Cassini-Soldner map projection has been used for over one hundred years and shall continue to be used for cadastral surveys in the new geodetic frame.
- 4.3.2 The mapping equations are as given in Richardus and Adler, 1974 and the formulas to derive projected Easting and Northing coordinate are as follows:

(a) Forward Computation

$$E = FE + v[A - T(A3/6) - (8 - T + 8C)T * A5/120]$$

$$N = FN + M - Mo + v * \tan\phi[A2/2 + (5 - T + 6C)A4/24]$$

where,

N, E = Computed Cassini coordinate

FE, FN = Cassini State origin coordinate

A = $(\lambda - \lambda_o)\cos\phi$

where,

λ = Longitude of computation point

λ_o = Longitude of state origin

ϕ = Latitude of computation point

T = $\tan 2\phi$

C = $\frac{e^2}{(1-e^2)} \cos^2\phi$

V = Radius of curvature in prime vertical

$= \frac{a}{(1-e^2 \sin^2\phi)^{1/2}}$

$$\begin{aligned}
M &= \text{Meridional arc distance} \\
&= a[1 - e^2/4 - 3e^4/64 - 5e^6/256 - \dots] \phi - (3e^2/8 + \\
&\quad 3e^4/32 + 45e^6/1024 + \dots) \sin 2\phi + (15e^4/256 \\
&\quad + 45e^6/1024 + \dots) \sin 4\phi - (35e^6/3072 + \\
&\quad \dots) \sin 6\phi \dots]
\end{aligned}$$

with ϕ is in radians.

M_0 = the value of M calculated for the latitude of the chosen origin.

(b) Reverse Computation

$$\begin{aligned}
\phi &= \phi_1 - \frac{v_1 \tan \phi_1}{\rho_1} \left[\frac{D^2}{2} - (1 + 3T_1) \frac{D^4}{24} \right] \\
\lambda &= \lambda_0 + \left[D - T_1 \frac{D^3}{3} + (1 + 3T_1) T_1 \frac{D^5}{15} \right] / \cos \phi_1
\end{aligned}$$

where,

$$\begin{aligned}
\phi_1 &= \mu_1 + (3e_1^2/2 - 27e_1^4/32 + \dots) \sin 2\mu_1 + (21e_1^2/16 - \\
&\quad 55e_1^4/32 + \dots) \sin 4\mu_1 + (151e_1^4/96 + \dots) \sin 6\mu_1 + \\
&\quad (1097e_1^6/512 - \dots) \sin 8\mu_1 + \dots
\end{aligned}$$

$$\rho_1 = \frac{a(1-e^2)}{(1-e^2 \sin^2 \phi_1)^{3/2}}$$

$$v_1 = \frac{a}{(1-e^2 \sin^2 \phi_1)^{1/2}}$$

where,

$$e_1 = \frac{1-(1-e^2)^{1/2}}{1+(1-e^2)^{1/2}}$$

$$\mu_1 = \frac{M_1}{a(1-e^2/4 - 3e^4/64 - 5e^6/256 - \dots)}$$

$$\begin{aligned}
 M_1 &= M_o + (N - FN) \\
 &= M_o \text{ is the value of } M \text{ calculated for the latitude of the origin} \\
 T_1 &= \tan^2 \phi_1 \\
 D &= (E - FE)/v_1
 \end{aligned}$$

(c) Scale and Arc-to-Chord Correction for Cassini Projection

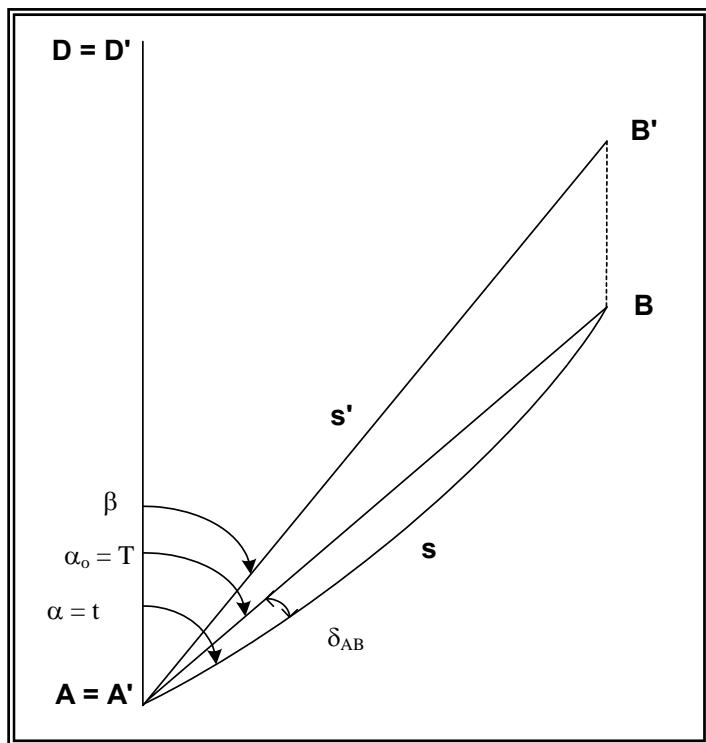


Figure 7: Scale and Arc-to-Chord

Refer to Figure 7,

$$\delta_{AB} = (t - T)'' = ((N_b - N_a)(E_b + 2E_a))/(6R^2 \cdot \sin 1'')$$

where,

$$\begin{aligned}
 N_a, N_b, E_a, E_b &= \text{Cassini Coordinate} \\
 (t - T)'' &= \text{Arc-to-Chord}
 \end{aligned}$$

Bearing correction:-

$$(\beta - \alpha)'' = -((\sin \alpha_0 \cdot \cos \alpha_0)/(6R^2 \cdot \sin 1'')) E_\mu^2$$

where,

$$E2\mu = (E2a + EaEb + E2b)$$

Linear correction:-

$$s' = s + [(\cos^2 \alpha_0)/(6R^2)]E^2_\mu s$$

Projected Coordinate on Cassini:-

$$E_b = E_a + s'.\sin\beta$$

$$N_b = N_a + s'.\cos\beta$$

4.4 Polynomial Function

The relationship between the MRSO coordinate and the Cassini Soldner_{Old} is defined by a series of polynomial function which make use of the coordinate of the origin of both projections for each state in Peninsular Malaysia. The following are the formulae used:

$$N(MRSO) = \Delta N(CAS) + N(OMRSO) + (R1+XA1+YA2+XYA3+X2A4+Y2A5)$$

$$E(MRSO) = \Delta E(CAS) + E(OMRSO) + (R2+XB1+YB2+XYB3+X2B4+Y2B5)$$

$$N(CAS) = \Delta N(MRSO) + N(OCAS) - (R1+XA1+YA2+XYA3+X2A4+Y2A5)$$

$$E(CAS) = \Delta E(MRSO) + E(OCAS) - (R2+XB1+YB2+XYB3+X2B4+Y2B5)$$

where

$N(MRSO)$: MRSO Coordinate in North Component

$E(MRSO)$: MRSO Coordinate in East Component

$N(CAS)$: Cassini Coordinate in North Component

$E(CAS)$: Cassini Coordinate in East Component

$\Delta N(CAS)$: Cassini Coordinate - Cassini Coordinate of Origin (North)

$\Delta E(CAS)$: Cassini Coordinate - Cassini Coordinate of Origin (East)

$\Delta N(MRSO)$: RSO Coordinate - RSO Coordinate of Origin (North)

$\Delta E(MRSO)$: RSO Coordinate - RSO Coordinate of Origin (East)

$X : \Delta N(CAS)/10000$ or $\Delta N(MRSO)/10000$

$Y : \Delta E(CAS)/10000$ or $\Delta E(MRSO)/10000$

All values of the coordinates must be in unit chains (use 0.11678249 as the multiplying factor to convert from metres)

4.5 Re-definition of State Origins in GDM2000 and GDM2000 (2009)

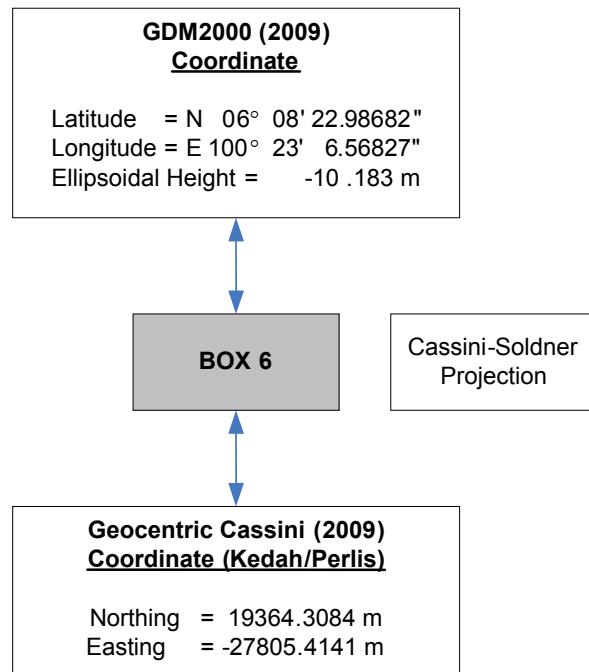
In order to prepare the geodetic infrastructure fully for the implementation of e-Kadaster, the re-definition of the nine (9) state cadastral origins need to be carried out. In this regard, JUPEM had successfully carried out GPS observations at all the origins in August and October of 2005 and the results are given in Table 5.

Table 5: Re-definition of State Cadastral Origins in GDM2000 and GDM2000 (2009) Coordinates

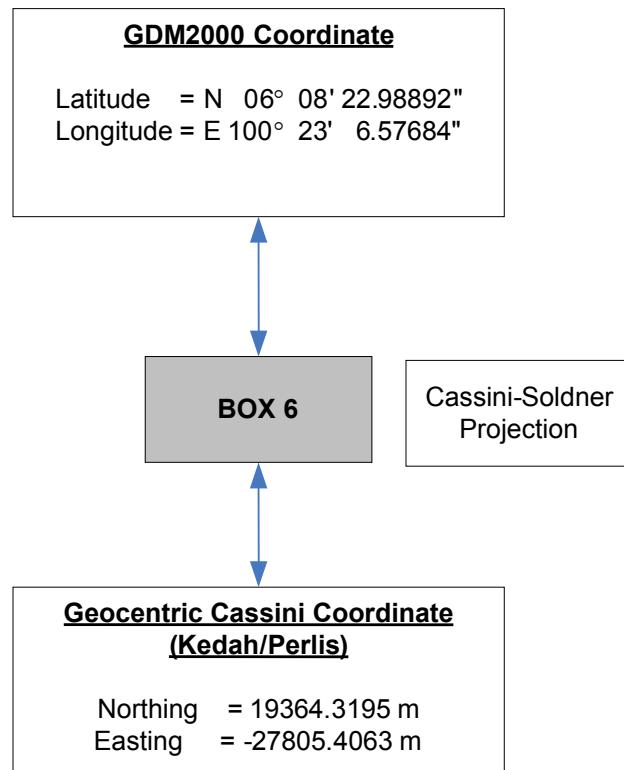
State	Station Location	GDM2000 (2009)		Cassini-Soldner	
		GDM2000			
		Latitude (North)	Longitude (East)	Northing	Easting
Johor	Gunung Belumut	2° 02' 33.20279"	103° 33' 39.83599"	0.000	0.000
		2° 02' 33.20196"	103° 33' 39.83730"		
Negeri Sembilan Melaka	Gun Hill	2° 42' 43.63412"	101° 56' 22.92628"	0.000	0.000
		2° 42' 43.63383"	101° 56' 22.92969"		
Pahang	Gunung Sinyum	3° 42' 38.69308"	102° 26' 04.60447"	0.000	0.000
		3° 42' 38.69263"	102° 26' 04.60772"		
Selangor	Bukit Asa	3° 40' 48.37751"	101° 30' 24.48130"	0.000	0.000
		3° 40' 48.37778"	101° 30' 24.48581"		
Terengganu	Gunung Gajah Trom	4° 56' 44.97144"	102° 53' 37.00068"	0.000	0.000
		4° 56' 44.97184"	102° 53' 37.00496"		
Pulau Pinang Seberang Perai	Fort Cornwallis	5° 25' 15.20204"	100° 20' 40.75188"	0.000	0.000
		5° 25' 15.20433"	100° 20' 40.76024"		
Kedah Perlis	Gunung Perak	5° 57' 52.81981"	100° 38' 10.93028"	0.000	0.000
		5° 57' 52.82155"	100° 38' 10.93860"		
Perak	Gunung Hijau Larut	4° 51' 32.64361"	100° 48' 55.46334"	0.000	0.000
		4° 51' 32.64488"	100° 48' 55.47038"		
Kelantan	Bukit Panau (Baru)	5° 53' 37.07908"	102° 10' 32.24004"	0.000	0.000
		5° 53' 37.07975"	102° 10' 32.24529"		

4.6 Test Examples

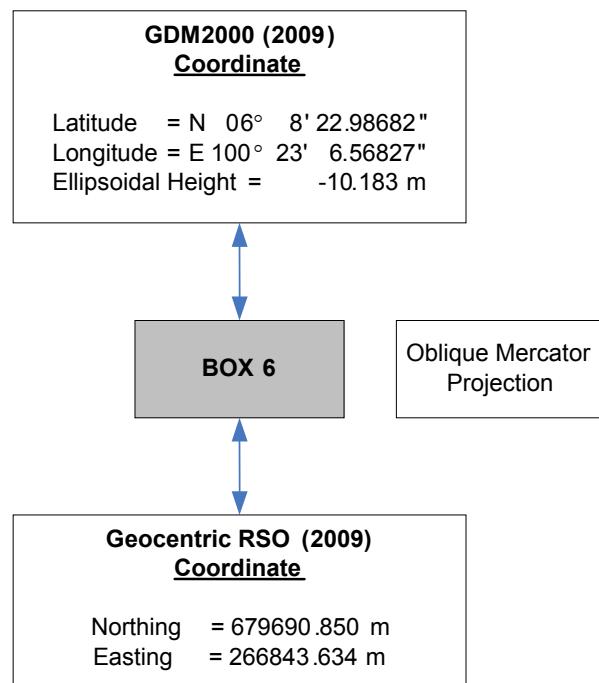
(i) GDM2000 (2009) to Geocentric Cassini (2009)



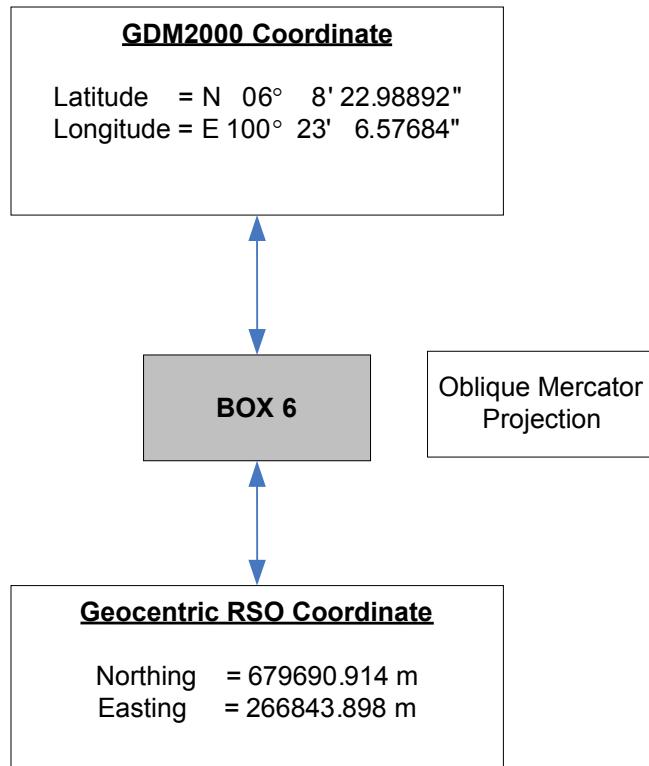
(ii) GDM2000 to Geocentric Cassini



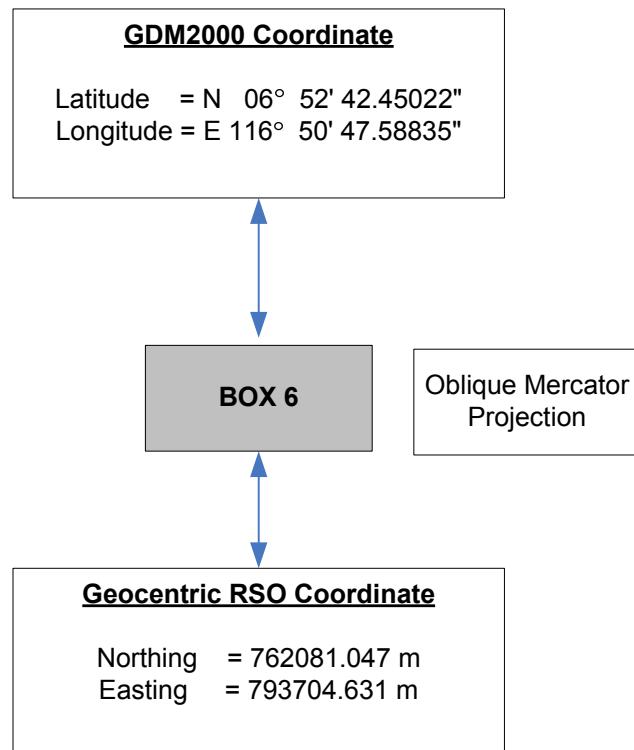
(iii) GDM2000 (2009) to Geocentric MRSO (2009)



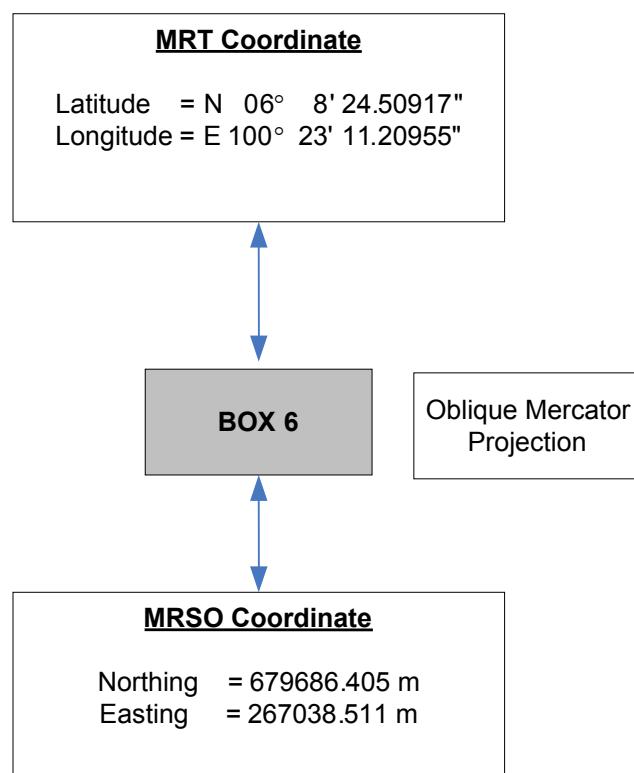
(iv) GDM2000 to Geocentric MRSO



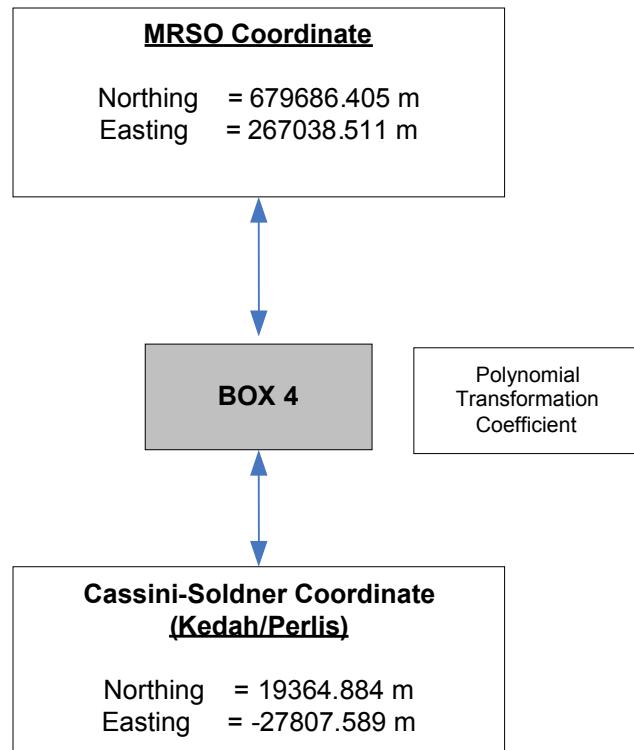
(v) **GDM2000 / GDM2000 (2009) to BRSO**



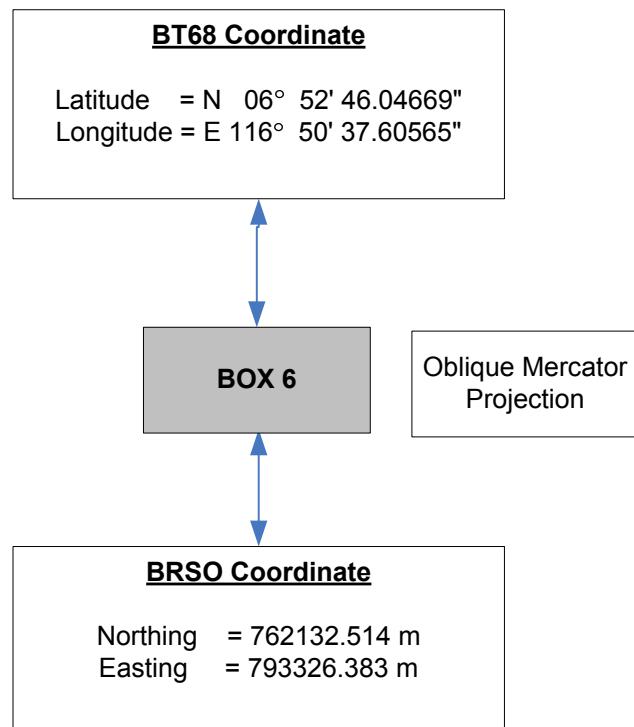
(vi) **MRT68 to MRSO**



(vii) MRSO to Cassini Soldner



(viii) BT68 to BRSO



5 CONCLUSION

- 5.1 The determination of a position requires the choice of a coordinate reference system. Until recently, an in-depth knowledge of coordinates and datums was generally confined to users with specialised knowledge in surveying. However, the development of techniques such as GPS has made available entirely new method of acquiring accurate coordinates. A situation now exists where it is common for a user acquiring data to be using a set of coordinates that is completely different to the one in which the data will be ultimately required.
- 5.2 For this reason and with the availability of various systems in Malaysia, JUPEM has produced various sets of datum transformation and map projection parameters to relate the different types of coordinate. This technical guide is designed for those dealing with coordinates as a practical solution to the problems that may be encountered with datums and map projections.
- 5.3 The parameter values relating to different coordinate reference systems are derived from standard coordinate conversion formulae, Bursa-Wolf transformation formulae and multiple regression model. In addressing the effects of plate tectonic motion as well as natural disasters particularly earthquakes, JUPEM will continue to monitor the geodetic infrastructures for any significant displacement and produce up-dated parameters to relate the various coordinates accurately and timely.

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